Application of Negative Feedback Control Algorithm in Controlling Nonlinear Rolling Motion of Ships

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Keywords: Ship Rolling Motion, Nonlinearity, Chaos, Negative Feedback Algorithm

Abstract: In order to describe the nonlinear phenomena and to find the regularity of ship rolling motion, the mathematical model of ship nonlinear rolling motion is built. The Lyapunov exponent curve is used to judge the chaos phenomena of ship nonlinear rolling motion. The nonlinear method of the negative feedback control algorithm is applied to control the chaos phenomena and to obtain the appropriate range of the negative feedback coefficient *k* so that the nonlinear phenomena are mitigated. The results show that the method can achieve a satisfactory control effect.

1. Introduction

The research of nonlinear phenomena of ship rolling motion includes: various nonlinear factors when ship is sailing, such as nonlinear restoring torque and damping torque, coupling effect of rolling and pitching, and of rolling and heaving, establishing nonlinear rolling coupling mathematical models, stability, frequency ratio, saturation, crossing and chaos phenomena of the roots of the coupling equation, the frequency of ship capsizing in the random waves, and so on. [1][2].

The nonlinear research of ship rolling motion dates back to the famous initiative research of scholar Euler, and then, scholar Krylov who developed the theory further[3][4]. With the improvement of the technique, domestic and oversea scholars have made great progresses and they adopted many theories, such as differential dynamic system theory, crossing theory and chaos theory.

In this paper, the mathematical model of ship nonlinear rolling motion is built up, the nonlinear method of the negative feedback control algorithm is used to solve the model, and different negative feedback coefficients are used to judge whether ship is in the chaotic state or not. Finally, the most appropriate range of the negative feedback coefficient is obtained. Thus, the chaos phenomena are gotten rid of and the necessary foundation for researching ship capsizing is laid.

2. Establishing Mathematical Model of Ship Nonlinear Rolling Motion

The existing research indicates that even though some basic factors are ignored, such as the coupling motion of six-DOF and the accurate determination of the hydrodynamic coefficients [5], the ship motion is very complex when is on sailing. However, the large rolling motion of nonlinear restoring torque and nonlinear damping torque should not be ignored. Nayfeh et al took the nonlinear restoring torque and damping torque into consideration and researched the stability and the complex problem of dynamics of rolling motion under the different slope of wave surfaces and encounter frequencies[6].In regular waves, the mathematical model of ship nonlinear rolling motion can be represented by the following differential equation[7][8]:

$$(I+I^*)\ddot{\varphi}+D(\dot{\varphi})+M(\varphi)=B_1-M(\dot{\alpha},\ddot{\alpha}) \tag{1}$$

Where I is the rolling inertia torque, N·m; I^* is the added rolling inertia torque, N·m; $D(\dot{\varphi})$ is the nonlinear damping torque, N·m; $M(\varphi)$ is the nonlinear restoring torque, N·m; B_1 is the fixed heeling torque, N·m; φ is the rolling angle, (°); $M(\dot{\alpha},\ddot{\alpha})$ is the wave disturbance torque, N·m; α is the encounter slope of wave surfaces, (°) , and can be defined as

$$\alpha = \alpha_m \cos \omega t \tag{2}$$

Where α_m is the maximum slope of wave surfaces, (°) ; ω is the encounter frequency, rad/s; t is the time variable, s. Because the chaos phenomena are extremely sensitive to external disturbances, just considering the influence of damping torque and restoring torque, $M(\dot{\alpha},\ddot{\alpha})$ can be represented as

$$M(\dot{\alpha},\ddot{\alpha}) = -f_1\alpha_m\omega \sin \omega t - f_2\alpha_m\omega^2 \cos \omega t \tag{3}$$

Where $-f_1\alpha_m\omega \sin \omega t$ is the restoring disturbance torque and $-f_2\alpha_m\omega^2 \cos \omega t$ is the damping disturbance torque. Inserting Eq.(3) into Eq.(1) ,we can derive:

$$(I+I^*)\ddot{\varphi} + D(\dot{\varphi}) + M(\varphi) = B_1 + f_1\alpha_m\omega \sin \omega t + f_2\alpha_m\omega^2 \cos \omega t \tag{4}$$

$M(\varphi)$ can be represented as:

$$M(\varphi) = \omega_0^2\varphi + \alpha_3\varphi^3 + \alpha_5\varphi^5 + \ldots \tag{5}$$

Where ω_0^2 is the linear coefficient of restoring torque, N·m; α_3 is the cubic coefficient; α_5 is the coefficient of fifth power.

$D(\dot{\varphi})$ can be represented as:

$$D(\dot{\varphi}) = 2\mu_1\dot{\varphi} + \mu_3\dot{\varphi}^3 \tag{6}$$

Where $2\mu_1$ is the linear coefficient of damping torque and μ_3 is the cubic coefficient.

$$B_1 = \omega_0\varphi_s + \alpha_3\varphi_s + \alpha_5\varphi_s \tag{7}$$

Where φ_s is the fixed heeling angle. Thus, B_1 can be considered as a constant.

Substituting Eqs.(5) -(7) in Eq.(4) , the mathematical model can be arranged and simplified as

$$(I+I^*)\ddot{\varphi} + 2\mu_1\dot{\varphi} + \mu_3\dot{\varphi}^3 + \omega_0^2\varphi + \alpha_3\varphi^3 + \alpha_5\varphi^5 = f_1\alpha_m\omega \sin \omega t + f_2\alpha_m\omega^2 \cos \omega t \tag{8}$$

Eq.(8) is simplified further as

$$\ddot{\varphi} + m_1\dot{\varphi} + m_2\dot{\varphi}^3 + n_1\varphi + n_2\varphi^3 + n_3\varphi^5 = A_1\alpha_m\omega \sin \omega t + A_2\alpha_m\omega^2 \cos \omega t \tag{9}$$

Where $m_1 = \frac{2\mu_1}{I+I^*}$, $m_2 = \frac{\mu_3}{I+I^*}$, $n_1 = \frac{\omega_0^2}{I+I^*}$, $n_2 = \frac{\alpha_3}{I+I^*}$, $n_3 = \frac{\alpha_5}{I+I^*}$, $A_1 = \frac{f_1}{I+I^*}$, and $A_2 = \frac{f_2}{I+I^*}$.

Taking a test ship model for example, its parameters are listed in Table 1.

Table 1. Parameters of the test model

parameters	m_1	m_2	n_1	n_2	n_3	A_1	A_2
proportions	0.3500	0.0222	3.2400	-4.5250	0.8780	0.5040	–4.6656

Inserting the parameters into Eq.(9) ,we can have

$$\begin{aligned}&\ddot{\varphi} + 0.3500\dot{\varphi} + 0.0222\dot{\varphi}^3 + 3.240\varphi - 4.5250\varphi^3 + 0.8780\varphi^5 \\ &= 0.5040\alpha_m\omega \sin \omega t - 4.6656\alpha_m\omega^2 \cos \omega t\end{aligned} \tag{10}$$

3. Judging Chaos Phenomena of Ship Nonlinear Rolling Motion

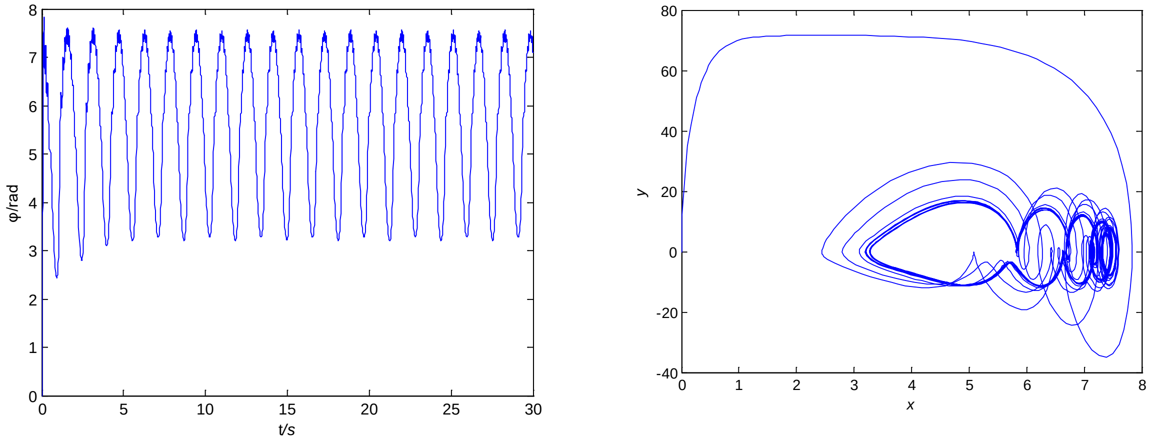
The principal characteristic of chaos system is that it is extremely sensitive to initial values. As time going on, traces generated by the little difference between the two initial values will disperse in the index way[9].

Duffing equation is a typical nonlinear equation and is widely used in detecting weak signal of chaos system.Eq.(10) is similar to Duffing equation. Now whether the chaos phenomena appear in the ship model is analyzed.

Letting $x=\varphi$, $y=\dot{\varphi}$ and $z=\omega t$, Eq.(10) can be represented as

$$\begin{cases} \dot{x}=y \\ \dot{y}=-3.240x+4.5250x^3-0.8780x^5-0.3500y-0.0222y^3+0.5040\alpha_m\omega\sin z-4.6656\alpha_m\omega^2\cos z \\ \dot{z}=\omega \end{cases} \quad (11)$$

When $\alpha_m=1.38$ and $\omega=1$, the initial condition (x_0,y_0,z_0)=(0, 0,1), simulation time is 30 seconds, the frequency spectrum diagram and phase-space diagram of the nonlinear system are shown in Fig.1.



(a) Frequency spectrum diagram (b) Phase-space diagram

Fig.1 Frequency spectrum diagram and phase-space diagram($\alpha_m=1.38$, $\omega=1$)

We find that with the increase of α_m, the rolling angle of ship motion becomes bigger, the dispersion phenomena of system are more obvious, and the rolling angle of ship motion becomes bigger correspondingly. These results show that the degree of dispersion becomes bigger. By analyzing Fig.1(b), the conclusion that the system of nonlinear rolling motion of ship is a chaotic system can be drawn.

4. Negative Feedback Controlling of Ship Nonlinear Rolling Motion

4.1 Constructing Negative Feedback Control Model.

The state of chaos can be controlled by a negative feedback coefficient. A negative feedback control parameter is constructed and a negative feedback coefficient is introduced. The control parameter, which is designed by the conventional back-stepping method[10], will counteract the nonlinear term in the system.

If the controlled system is complicated, there will be more unknown parameters, so a simple Lyapunov function is constructed, control processes are simplified, the number of unknown parameters is reduced, and the control parameters which don't counteract nonlinear terms are designed. Based on a simple robust control method, the parameter x is controlled in this paper, and then this algorithm is of certain robustness.

In Eq.(11), introducing the negative feedback parameter u into $\dot{x}=y$, it can be hypothesized as

$$\dot{x}=y+u \tag{12}$$

u can be expressed as

$$u=kx+v \tag{13}$$

Where k is a negative feedback coefficient; v is the reference value that is without the loss of generality, so hypothesize v = 0. Set $\alpha_m=1$, thus, $\dot{x}=y$ will be turned into

$$\dot{x}=kx+y \tag{14}$$

Eq.(11) will be represented as

$$\begin{cases}\dot{x}=kx+y \\ \dot{y}=-3.240x+4.5250x^3-0.8780x^5-0.3500y-0.0222y^3+0.5040\omega\sin z-4.6656\omega^2\cos z \\ \dot{z}=\omega\end{cases} \tag{15}$$

Because the primary system can be controlled, an appropriate k has to be found and used to make the primary system into the state of stability. The state of chaos can be judged from the general knowledge that the Lyapunov exponent is greater than zero directly. If all Lyapunov exponents are negative, the system will be in the state of stability. If positive Lyapunov exponents occur, the system will be in the state of chaos[11]. In this case, appropriate measures could be taken to control the chaos system.

4.2 Calculating Negative Feedback Coefficient.

Because the state of system can be judged from the Lyapunov exponent, the Lyapunov exponent curve will change with the change of k. If all Lyapunov exponents are negative, the system will be in the state of stability, namely, the range of k is appropriate. This paper uses the method of Jacobian matrix to solve k.

Eq.(15) is turned into the form of Jacobian matrix as

$$J=\begin{pmatrix} k & 1 & 0 \\ -A & -B & -C \\ 0 & 0 & 0 \end{pmatrix} \tag{16}$$

Where $A=4.39(x^2-1.546)^2-7.25$, $B=0.066y^2+0.35$, $C=0.5040\cos z-4.665\sin z$.

Let det(λE-J)=0， where λ is an eigenvalue and E is the third-order identity matrix. The determinant can be shown as

$$\begin{vmatrix} \lambda-k & -1 & 0 \\ A & \lambda+B & C \\ 0 & 0 & \lambda \end{vmatrix}=0 \tag{17}$$

Solving Eq.(17) is as follows:

$$\lambda(\lambda-k)(\lambda+B)+\lambda A=0 \tag{18}$$

Eq.(18) has three eigenvalues:

$$\lambda_1=0, \lambda_{2,3}=-\frac{B-k}{2}\pm\sqrt{(kB-A)+\frac{(B-k)^2}{4}} \tag{19}$$

In order to eliminate state of chaos, all Lyapunov exponents should be negative, namely, Eq.(19) has real roots and λ is negative.

$$\max\left\{-\frac{B-k}{2}\pm\sqrt{(kB-A)+\frac{(B-k)^2}{4}}\right\}\leq 0 \tag{20}$$

In order to make the equation (20) hold, it can be written into the following inequality group:

$$\begin{cases}(kB-A)+(B-k)^2/4\geq 0\\(B-k)/2\geq 0\\kB-A\leq 0\end{cases} \tag{21}$$

For the inequality $(k+B)^2\geq 4A$, if $A<0$, the inequality holds, thus

$$k\in R \tag{22}$$

If $A\geq 0$, we have

$$k\geq -B+2\sqrt{A} \quad \text{or} \quad k\leq -B-2\sqrt{A} \tag{23}$$

Substituting *A* and *B* in the inequalities (23), we can get

$$k\geq 2\sqrt{4.39(x^2-1.546)^2-7.25}-0.066y^2-0.35$$

or

$$k\geq -2\sqrt{4.39(x^2-1.546)^2-7.25}-0.066y^2-0.35$$

namely,

$$k\geq -0.35 \tag{24}$$

or

$$k\leq -0.35 \tag{25}$$

For $k\leq B$, inserting B into it, we get

$$k\leq 0.35 \tag{26}$$

For $k\leq A/B$ (where $A\geq 0$, $B\geq 0$), inserting *A* and *B* into it, we get

$$k\leq\frac{4.39(x^2-1.546)^2-7.25}{0.066y^2+0.35}$$

namely,

$$k\leq 0 \tag{27}$$

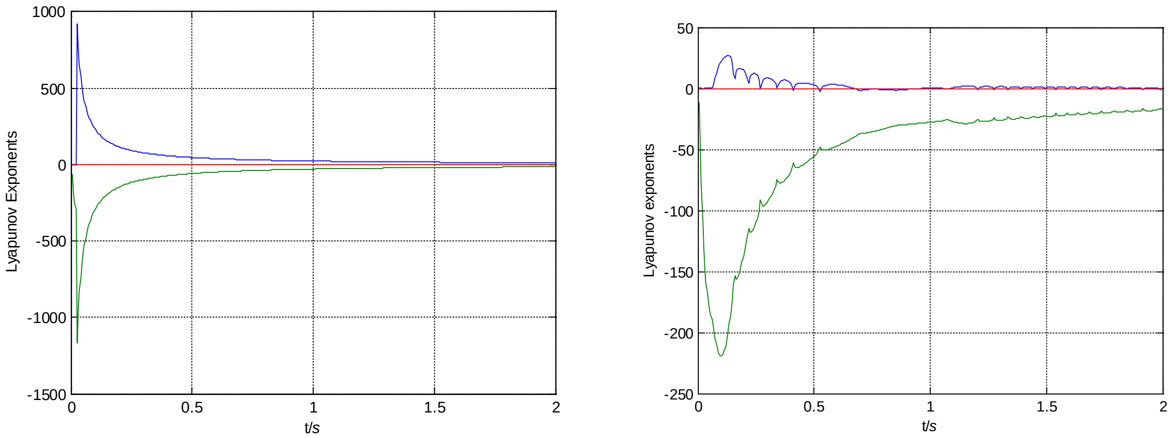
Combining the results from the above inequalities (24)-(27), we can get the appropriate range of *k*.

$$k\leq -0.35 \tag{28}$$

4.3 Controlling the State of Chaos.

The purpose of controlling the state of chaos is to eliminate the chaos phenomena and to reduce the amplitude. For different *k*, MATLAB is used to draw Lyapunov exponent diagrams as shown in Fig.2. When $k=-0.35$, there exists an exponent curve that gradually becomes steady at zero, which indicates that the system just breaks away from the chaos phenomena. When $k>-0.35$, there exists an exponent curve that is greater than zero, which indicates that chaos phenomena occur in the system. The chaos phenomena are more obvious with *k* becoming bigger.

With *k* becoming smaller, the system will be more stable. These simulation results reveal that using different k, we can get different Lyapunov exponent diagrams, such as Fig.2(b), so we can judge whether the system is under the state of chaos or breaks away from it.



(a) $k > -0.35$ (b) $k = -0.35$

Fig.2 Lyapunov exponent diagram

When $k < -0.35$, all exponent curves are less than zero, which indicates that the system completely breaks away from the chaos phenomena. The result validates that the negative feedback coefficient k has good controlling result.

5. Conclusions

Based on the Lyapunov exponent curve, whether the system in the state of chaos or not is judged, the nonlinear phenomena of ship swaying motion (especially, the nonlinear rolling motion) are analyzed, the mathematical model of ship nonlinear rolling motion is established, the chaos theory is used to explain this phenomena, the negative feedback algorithm is adopted to control the state of chaos, the range of the negative feedback coefficient is solved, MATLAB is used to draw different Lyapunov exponent diagrams, and finally the system breaking away from the state of chaos is validated. By analyzing and validating the coefficient, it can be found that the coefficient has good controlling results. This paper provides further analysis and research for eliminating capsizing accidents caused by large rolling motion.

Acknowledgements

This research was supported by Zhejiang Provincial Natural Science Foundation of China under Grant No. LQ18E070004 and Grant No. LQ15F030007.

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